

**SIMULATION APPROACHES IN  
TRANSPORTATION ANALYSIS**

*Recent Advances and Challenges*

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**SIMULATION APPROACHES IN  
TRANSPORTATION ANALYSIS**  
*Recent Advances and Challenges*

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# SIMULATION APPROACHES IN TRANSPORTATION ANALYSIS: Recent Advances and Challenges

Ryuichi Kitamura and Masao Kuwahara

## **Preface**

Achieving efficient, safe, and convenient urban automotive transportation has been the primary concern of transportation planners, traffic engineers, and operators of road networks. As the construction of new roadways becomes increasingly difficult and, at the same time, as the adverse environmental impacts of automotive traffic are more critically assessed, and as the depletion of fossil fuels and global warming loom as serious problems, it is now imperative that effective traffic control strategies, demand management schemes and safety measures be expeditiously implemented.

The advent of advanced information and telecommunications technologies and their application to transportation systems have expanded the range of options available in managing and controlling network traffic. For example, providing individualized real-time information to drivers is now almost reality. Evolving Intelligent Transport Systems (ITS) technology is making it possible to link the driver, vehicle and road system by exchanging information among them, calling for the development of new traffic control strategies. It is in this context that transport simulation is emerging as the key concept in traffic control and demand management.

Motivated by this line of thought, the International Symposium on Transport Simulation was held in Yokohama, Japan, in August 2002. It aimed at providing a forum where groups of researchers who are engaged in

cutting-edge research in transport simulation would gather from all over the world, and exchange their results, discuss research issues, and identify future directions of development. Specifically, it was envisaged that the Symposium would contribute to the development and application of transport simulation methods by

- introducing state-of-the-art transport simulation models, methodologies, and examples of their applications,
- identifying short-term and long-term research issues,
- assessing promising application areas for simulation, and
- evaluating the applicability of transport simulation methods to other research areas.

It was also hoped that this Symposium would aid in establishing a worldwide network of researchers involved in transport simulation.

A total of 19 papers were presented at the Symposium. While some were concerned with specific simulation model systems and their application (most of the major simulation model systems available were represented at the Symposium), others addressed various research issues in transport simulation. This volume contains a set of papers selected from those presented at the Symposium.

This book is divided into four parts. Part I comprises four papers that represent simulation models for dynamic network assignment. The first paper by Florian *et al.*, which was the keynote speech at the Symposium, offers a concise review of existing dynamic network simulation models and an overview of issues involved in simulation-based dynamic traffic assignment. A successful application example to a medium-size network is then presented. The second paper by Liu presents the dynamic traffic network simulator, DRACULA, whose unique features include the representation of day-to-day variation in demand and drivers' learning. The following paper by Barcelo and Casas presents the microscopic traffic simulator, AIMSUN, describes car following, lane changing and other elements of the model system, and discusses its application to dynamic network simulation. The last paper of Part I by Toledo *et al.* describes the microscopic traffic simulation tool, MITSIMLab, detailing its representation of driver behavior, touching on calibration and validation issues, and demonstrating a Stockholm, Sweden, case study results.

Part II contains three papers that are concerned with the development and application of transport simulation. The first paper by Horiguchi and

Kuwahara assesses the status of simulation model application in Japan, describes the ongoing effort toward standardized model verification and validation, and report on the establishment of a forum for information exchange in Japan. The second paper by Takayama and Nakayama discusses the estimation of an origin-destination matrix in conjunction with network simulation, where paths on the network are assumed to follow an absorbing Markov chain process, whose parameters are estimated using genetic algorithms. In the third paper by Asakura *et al.*, mobile communications technology is applied to acquire space-time trajectory data from cellular phone holders, and the resultant data are applied to simulate how the participants of a sports event disperse and travel toward their respective destinations after the event is over.

Estimating and representing the dynamics of traffic flow and individual vehicle movement is the common concern of the four papers in Part III. In the first paper, Schreckenberg *et al.* attempt to combine, as “an online-tool,” information from real-time traffic data from detectors and results of microscopic traffic simulation to determine the state of traffic throughout the German autobahn network. Young and Weng discuss issues involved in simulating parking in urban area, addressing the microscopic representation of vehicle movement in parking facilities, drivers’ decision making including route choice, and the interaction among data collection, model accuracy and model validity. The third paper by Akahane *et al.* offers a historical summary and assessment of the development of traffic simulation models in Japan, describing the evolution of the representation of vehicle dynamics, path definition methodologies, and improvement in computational efficiency. In the last paper of Part III, Nanthawichit *et al.* propose a methodology to combine probe data and conventional detector data to estimate the state of traffic on roadway segments with improved accuracy, through the application of Kalman filters.

As its title suggests, Part IV is concerned with the representation of user behavior. The first paper by Harata addresses the issue of consistency between traffic simulation models and travel behavior choice theory, and examines specifically dynamic route choice models and time-of-day choice models. Following this, Morikawa *et al.* critically examine the traditional assumptions of shortest-path choice and user equilibrium based on perfect information, through network assignment with imperfect information. In the last paper of the volume, Kitamura *et al.* present PCATS, a micro-simulator of individuals’ daily travel, which produces trip demand along a continuous time axis, and illustrate its application to the analysis of TDM measures and to long-term demand forecasting along with a dynamic network simulator, DEBNetS.

As these chapters demonstrate, transport simulation has become a powerful tool in both research and practice. It can be a practical tool for the real-time forecasting of future traffic status on road networks and evaluation of the effectiveness of alternative traffic control measures. It can be applied in the assessment of the effectiveness of alternative TDM measures, or in the selection and implementation of a variety of ITS schemes now being developed. It is hoped that this volume will aid in further development of transport simulation models and their prevalent adoption as a practical tool in traffic control and transportation planning.

A large number of individuals contributed to the organization of the Symposium and the editing of this book. In particular, we note the efforts by Drs. Akira Kikuchi, Hiroyuki Oneyama and Toshio Yoshii, who contributed tremendously throughout this project. Special thanks go to Ms. Kiyoko Morimoto; we owe the success of the Symposium to her bookkeeping and management skills. We also thank the speakers, presenters at the demo sessions, and the audience at the Symposium. Finally, we dedicate this book to those researchers and practitioners whose endeavors are contributing to better urban transportation in many significant ways.

# APPLICATION OF A SIMULATION-BASED DYNAMIC TRAFFIC ASSIGNMENT MODEL

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## ABSTRACT

The evaluation of on-line intelligent transportation system (ITS) measures, such as adaptive route-guidance and traffic management systems, depends heavily on the use of faster than real time traffic simulation models. Off-line applications, such as the testing of ITS strategies and planning studies, are also best served by fast-running traffic models due to the repetitive or iterative nature of such investigations. This paper describes a simulation-based, iterative dynamic-equilibrium traffic assignment model. The determination of time-dependent path flows is modeled as a master problem that is solved using the method of successive averages (MSA). The determination of path travel times for a given set of path flows is the network-loading sub-problem, which is solved using the space-time queuing approach of Mahut. This loading method has been shown to provide reasonably accurate results with very little computational effort. The model was applied to the Stockholm road network, which consists of 2100 links, 1,191 nodes, 228 zones, representing and 4,964 turns. The results show that this model is applicable to medium-size networks with a very reasonable computation time.

*Keywords – dynamic traffic assignment, method of successive averages, traffic simulation, queuing models*

## INTRODUCTION

The functional requirements of a dynamic traffic assignment (DTA) model for ITS applications may be subdivided into two major modes of use: off-line and on-line. The off-line use of DTA is for the testing and evaluation of a wide variety of ITS measures before they are implemented in practice. In particular, iterative approaches to dynamic assignment that approximate (dynamic) user equilibrium conditions are generally restricted to off-line use due to the high computation times involved. The resulting assignments can also be interpreted as imitating drivers' adaptation over time to changes in network topology or control, including the implementation of ITS measures. Due to the high number of iterations usually required, such applications are ideally suited for traffic models that have low computational requirements. On-line DTA can be used within a system that monitors and manages the network in real time. DTA and the embedded traffic models can play a key role in providing short-term forecasts of the system state that are used by adaptive traffic management, control and guidance systems. Due to the need to provide feedback in real time, on-line DTA poses rather stringent demands on the embedded models for maintaining low computation times.

The need to model the time varying network flow of vehicles for ITS applications has generated many contributions for the solution of dynamic traffic assignment methods. These contributions are varied and have been motivated by different methodological approaches. They may be classified according to the modeling paradigm underlying the temporal traffic model. In order to provide a common terminology to the various models, it is convenient to refer to two main components of any dynamic traffic model: the route-choice mechanism and the network-loading mechanism. The latter is the method used to represent the evolution of the traffic flow over the links of the network once the route choice has been determined.

Perhaps the most popular dynamic traffic models today are those based on the representation of the behavior of each driver regarding car following, gap acceptance and lane choice. These are micro-simulation models such as CORSIM ([http://www.fhwa-tsis.com/corsim\\_page.htm](http://www.fhwa-tsis.com/corsim_page.htm)), INTEGRATION

(Van Aerde, 1999), AIMSUN2 (Barceló et al, 1994) (<http://www.tss-bcn.com>), VISSIM (<http://www.ptv.de>), PARAMICS (<http://www.quadstone.com>) and DRACULA (<http://www.its.leeds.ac.uk/software/dracula/>). MITSIM (Yang, 1997) (<http://web.mit.edu/its/products.html>) is an academic research model that has been used in several studies in Boston, Stockholm and elsewhere.

There are many other micro-simulation models developed in universities and industrial research centers that use the same basic approach. The route choice in a micro-simulation model is either predetermined or computed while the loading of the network is being carried out. Essentially, a micro-simulation model aims to provide the traffic flows composed of individual vehicles in one network-loading step. As micro-simulation models are built by using many stochastic choice mechanisms, their proper use requires the replication of runs. The successful use of micro-simulations is commonly limited to relatively small size networks. Their application has been hindered for medium-to-large networks by the relatively high computation time and effort required for a proper model calibration. Usually, there are many parameters involved. Choosing appropriate parameter values is a relatively complex task since each computer implementation for a micro-simulation uses a large number of heuristic rules that are added to the basic car-following mechanism. A thorough understanding of how these parameter choices influence the results, when using given software package, is essential for a successful application. Nevertheless, micro-simulation models are popular and their use is enhanced by traffic animation graphics that capture the attention of non-technical staff.

The aim of handling larger networks with reasonable computational times has led to the development of so-called “mesoscopic” approaches to traffic simulation, which are less precise in the representation of traffic behavior but are less cumbersome computationally. The aim is to obtain a traffic representation that still captures the basic temporal congestion phenomena, but models the traffic dynamics with less fidelity. One of the earliest examples of such an approach is CONTRAM (Leonard et. al., 1989) ([www.contram.com](http://www.contram.com)) which is a commercially available package that has been used in England and elsewhere in Europe.

Recently, the development of mesoscopic simulation models for off-line dynamic traffic assignment has become an area of significant research

activity, as witnessed by the United States Federal Highway Administration Dynamic Traffic Assignment Project (<http://www.dynamictrafficassignment.org>). The development of DYNASMART (Mahmassani et al., 2001) and DYNAMIT (Ben-Akiva et al., 1998) (<http://web.mit.edu/>) are two significant developments. These mesoscopic models provide a path choice mechanism and a network loading method based on simplified representations of traffic dynamics. While CONTRAM represents traffic with continuous flow, as it has its roots in static traffic assignment models, DYNASMART and DYNAMIT move individual vehicles. CONTRAM and DYNAMIT provide an iterative scheme for the emulation of dynamic user equilibrium, where all cars within the same departure interval for a given origin-destination pair experience the same travel time (approximately). The approach taken in DYNAMIT is to provide an “a priori” path choice and path set by using models based on random choice utility theory. Another approach to the network loading algorithm is that based on cellular automata theory (Nagel and Schreckenberg, 1992), which has been implemented in the TRANSIMS software (<http://transims.tsasa.lanl.gov>), developed recently by the Los Alamos National Laboratories in the USA. In this approach, the route choice is predetermined for each traveler and the network loading method loads the vehicles on a network where each lane of a link is divided into cells of equal size. The advance of vehicles is carried out by using local rules for each vehicle that determine the next cell to be occupied and the speed of the vehicle.

Other dynamic traffic assignment models have their roots in macroscopic traffic flow theory developed during the 1950's (Lighthill and Whitham, 1955) (Richards, 1956). The work of Papageorgiou (1990) led to the development of the METACOR (Diakakis and Papageorgiou, 1996) and METANET (Messmer et al., 2000a), which has been used for the development of an iterative dynamic traffic assignment method (Messmer et al., 2000b). The route choice in this model is carried out by splitting proportions at nodes of the network, where only two arcs can originate at a given node. The network loading method is based on a second order (p.d.e.) traffic flow model.

Another line of research is that of analytical dynamic traffic assignment models, which has its roots in the mathematical programming approach to

static network equilibrium models. This area is not covered in this contribution.

The dynamic assignment model presented in this paper is based on a traffic simulation model that was designed to produce reasonably accurate results with a minimum number of parameters and a minimum of computational effort (Mahut, 2000, Astarita et al., 2001)). However, the underlying structure of the model has more in common with microscopic than with mesoscopic approaches, as it is designed to capture the effects of car following, lane changing and gap acceptance. The simulation is a discrete-event procedure and moves individual vehicles. Unlike discrete-time microscopic simulation models, where the computational effort per link is proportional to the total vehicle-seconds of travel, the computational effort per link required by this model is strictly proportional to the number of vehicles to pass through it, regardless of their travel times. As a result, the relative efficiency of this approach compared to microscopic methods increases with the level of congestion.

Another special property of this model is that the traffic dynamics are modeled without the (longitudinal) discretization of links into segments or cells. As a result the procedure only explicitly calculates the time at which each vehicle crosses each node on its path. This leads to a drastic reduction in computational effort relative to microscopic discrete time approaches, where the computational effort is a function of the total travel time experienced by the drivers.

The paper is structured as follows. The next section is dedicated to the exposition of approaches to dynamic traffic assignment. The third section is dedicated to the description of the network loading method developed; the algorithm for the dynamic traffic assignment, which combines the route-choice mechanism with the network loading method, is presented in the fourth section. Applications of the model are then given and some conclusions end the paper.

## **DYNAMIC TRAFFIC ASSIGNMENT**

Two different approaches are commonly used to emulate the path choice behavior of drivers: dynamic assignment *en route* and dynamic *equilibrium*

assignment. In this work, the approach taken is to seek an approximate solution to the dynamic equilibrium conditions.

### **En-Route Assignment**

In the en route assignment problem, the routing mechanism is a set of behavioral rules that determine how drivers react to information received en route. Information may be available at discrete points in time (e.g. radio broadcasts), discrete points in space (e.g. variable message signs), or be continuously available in both space and time (e.g. traffic conditions visible to the driver). Some information may only be available to a certain class of vehicles; e.g., those equipped with vehicle guidance systems. Typically, the choice of what information is provided to the drivers, i.e., the information *strategy*, is an exogenous input. Moreover, how drivers respond to information is also an exogenous input and may involve one or more parameters, such as the ‘penetration rate’. The output is the resulting (time-dependent) path choices given the time-dependent origin-destination demand. Another input to this problem is a suitable pre-trip assignment, i.e., path choices that represent the “do nothing” alternative and which are followed in the absence of any en route information. In many cases, an equilibrium assignment (discussed below) is used for this purpose.

En route assignment thus only requires running a single dynamic (time-dependent) loading of the demand onto the network over the time period of interest. If the information strategy or the driver response strategy is parameterized, it may be possible to design an iterative algorithm to determine the optimal values of such parameters.

### **Equilibrium Assignment**

In the equilibrium assignment problem, only pre-trip path choices are considered. However, the path choices are modelled as a decision variable and the objective is to minimize each driver’s travel time. All drivers have perfect access to information, which consists of the travel times on all paths (used and unused) experienced on the previous iterations. All drivers furthermore attempt to minimize their own travel times, and the solution algorithm takes the form of an iterative procedure designed to converge to these conditions. The solution algorithm used here consists of two main components: a method

to determine a new set of time-dependent path flows given the experienced path travel times on the previous iteration, and a method to determine the actual travel times that result from a given set of path flow rates. The latter problem is referred to as the “network loading problem”, and can be solved using any route-based dynamic traffic model. The algorithm furthermore requires a set of initial path flows, which are determined by assigning all vehicles to the shortest paths, based on free-flow conditions. The general structure of the algorithm is shown schematically in Figure 1.

The mathematical statement of the dynamic equilibrium problem is in the space of path flows  $h_k(t)$ , for all paths  $k$  belonging to the set  $K_i$  for an origin-destination  $i \in I$ , at time  $t$ . The time-varying demands are denoted  $g_i(t)$ . The path flow rates in the feasible region  $\Omega$  satisfy the conservation of flow and non-negativity constraints for  $t \in T_d$ , where  $(0, T_d)$  is the period during which the temporal demand is defined. That is

$$\Omega = h(t) : \sum_{k \in K_i} h_k(t) = g_i(t), i \in I; h_k(t) \geq 0$$

(1)

*for almost all  $t \in T_d$*

The definition of user optimal dynamic equilibrium is given by the temporal version of the static (Wardrop) user optimal equilibrium conditions, which are:

$$s_k(t) = u_i(t) \text{ if } h_k(t) > 0$$

$$s_k(t) \geq u_i(t) \text{ otherwise}$$

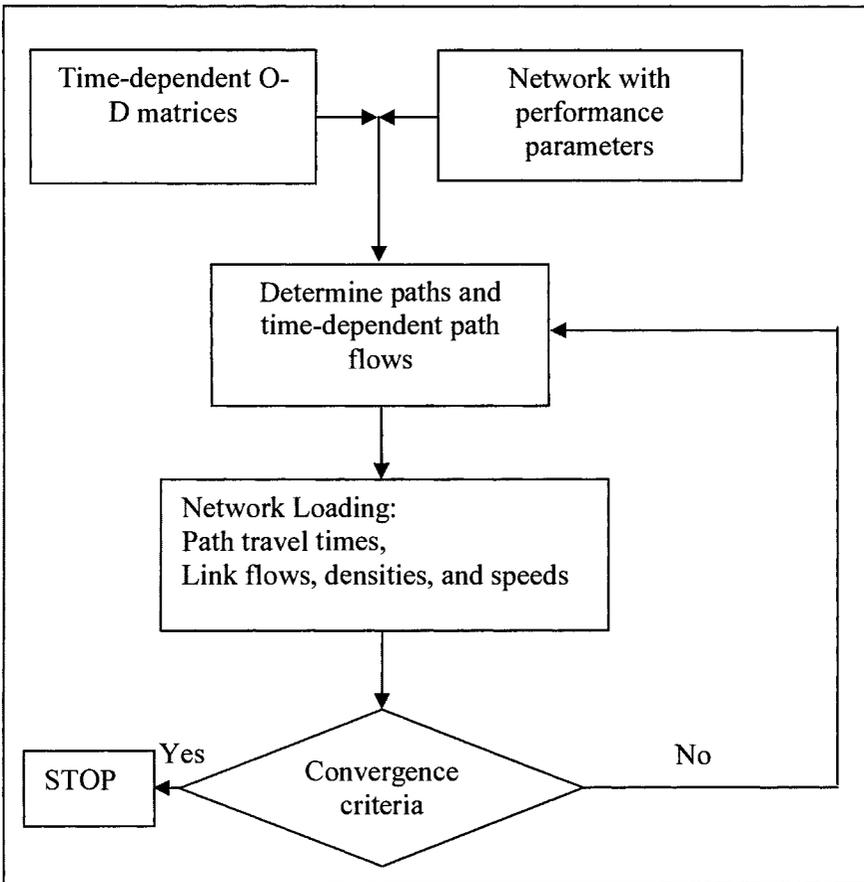
(2)

for all:  $k \in K_i, i \in I$ , for almost all  $t \in T_d$

where:  $h_k \in \Omega, u_i(t) = \min_{k \in K_i} \{s_k(t)\}$  for almost all  $t \in T_d$  and  $s_k(t)$  is the path travel time determined by the dynamic network loading. Friesz et al (1993) showed that these conditions are equivalent to a variational inequality problem, which is to find  $h^* \in \Omega$  such that

$$(S(h^*), h - h^*) \geq 0, \forall h \in \Omega$$

(3)



**Figure 1. Structure of solution algorithm to DTA problem.**

The continuous time problem (1)-(3) is usually solved by using some time discretization scheme.

## THE ALGORITHM

The solution approach adopted for solving the dynamic network equilibrium model (1)-(3) is based on a time discretization into discrete time periods

$\tau = 1, 2, \dots, \left\lfloor \frac{T_d}{\Delta t} \right\rfloor$ , where  $\Delta t$  is the chosen duration of a time interval. This

results in the discretized model

$$\begin{aligned}
 s_k^\tau &= u_i^\tau \text{ if } h_k^\tau > 0 \\
 s_k^\tau &\geq u_i^\tau \text{ if } h_k^\tau = 0 \\
 \text{for all } k \in k_i, i \in I, \tau &= 1, 2, \dots, \left\lfloor \frac{T_d}{\Delta t} \right\rfloor
 \end{aligned} \tag{4}$$

where the feasible set of time dependent flows  $h_k^\tau$  belong to

$$\begin{aligned}
 \Omega^\tau &= \{h_k^\tau : \sum_{k \in K_i} h_k^\tau = g_i^\tau, i \in I, \text{ all } \tau ; \\
 &h_k^\tau \geq 0, k_i, i \in I, \text{ all } \tau\}
 \end{aligned} \tag{5}$$

which can be shown to be equivalent to solving the discretized variational inequality.

$$\sum_{\tau} \sum_{k \in K} s_k^\tau(h^\tau) (h_k^\tau - h_k^\tau) \geq 0 \tag{6}$$

where  $K = \bigcup_{i \in I} k_i$  where  $h^\tau$  is the vector of path flows  $(h_k^\tau)$  for all  $k$  and  $\tau$ .

The path input flows  $h_k^\tau, k \in K$  are determined by the method of successive averages (MSA), which is applied to each O-D pair  $I$  and time interval  $\tau$ .

The initialization procedure consists of an incremental loading scheme that successively assigns partial sums of the demand for each interval  $\tau$  onto dynamic shortest paths. That is, the first demand increment,  $g_i^1, i \in I$  is loaded onto a dynamic shortest path based on free flow travel times; there the link travel times are updated and a new dynamic shortest path is computed for interval 2; the first and second demand increments  $(g_i^1, g_i^2)$  are loaded onto the first and second computed paths and so on.

Starting at the second iteration, and up to a pre-specified maximum number of iterations,  $N$ , the time-dependent link travel times after each loading are used to determine a new set of dynamic shortest paths that are added to the current set of paths. At each iteration  $n, n \leq N$ , the volume assigned as input flow to